Chapter 11: Inference for Distributions

- Still creating confidence intervals and performing significance tests
- will test for mean of single population (11.1)
- also will compare means of two populations (11.2)

11.1 - Inference for the Mean of a Population

CI and tests of significance for the mean μ of a normal population:

- are based on the distribution of the sample means
- sampling distribution has μ as its mean
- spread of sampling distribution depends on sample size and σ

In Chapter 10, we assumed a σ,

- we will not normally know σ so we will estimate
- this will change some of the details of our tests and CI
- the interpretations will be the same

Still need to meet the same two conditions:

- unbiased sample (SRS) - VERY important
- population is N(μ,σ) with both μ and σ unknown

Sample mean $\bar{x}$ has N(μ,σ/$\sqrt{n}$) and since we don't know σ, we use the sample standard deviation, which is called standard error: s.

Now: estimate $\sigma_{\bar{x}}$ using: $\frac{s}{\sqrt{n}}$, which we call the standard error of the sample mean $\bar{x}$ (SEM or SE$_{\bar{x}}$)

t distribution

when we know σ, we base confidence intervals and signif. tests on the one-sample z statistic. When we don't know σ, we substitute standard error s/$\sqrt{n}$ of $\bar{x}$ for its standard deviation σ/$\sqrt{n}$.

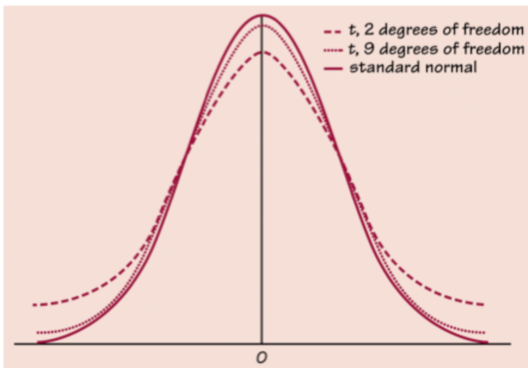
The distribution for the resulting "t" does not have a normal distribution, but a new distribution called a t distribution which has more tail probability and less central probability due to the larger estimated spread.

Instead of a one-sample z statistic and a normal distribution for calculating probabilities,

we will use a one-sample t statistic and the t distribution:

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$ has the t distribution with n-1 degrees of freedom

the degrees of freedom is directly related to sample size. As sample size increases, degrees of freedom increase and the t distribution approaches normal.



we write: t(k) for t distribution with k degrees of freedom

Good to know:

- when the sample size is large, there is very little variation caused by substituting s for σ

- when the sample size is less than 40, you must display your data and analyze it for any signs that the population it represents might not be normally distributed. Good displays and discussion points are:
  * box plot (good for checking outliers)
  * stemplot or dotplot (excellent for displaying smaller sets of data)
  * histogram (better for larger data sets)
  * normal probability plot (should use as a secondary check of normality when roughly linear - very good for assessing skewness)

- when the data is collected from a matched pairs design, the difference between the measures for each pair becomes the single observation for each subject and 1-sample t procedures are used.

- when calculating power or Type II error, use t* to solve for $\bar{x}$ in the Ho distribution, but then use z formula and normalcdf for calculating the probabilities in the Ha distribution.

Guide to Confidence Interval - unknown σ (1 sample t-interval)

purpose parameter population

1. I want to estimate the true mean ___________ of _________________.

2. Since σ is unknown, I will construct a 1 sample t-interval
   - **Condition 1**: need an unbiased sample
     * If SRS, then OK.
     * If random or not specified, then address the issue of potential bias in your results
   - **Condition 2**: need an approximately normal population
     * **n < 15** - display data, must be single peaked and approx. symmetrical. No skew or outliers. If there are, it must be mentioned as a major issue in generalizing your results to the population.
     * **n between 15 and 39** - display data, minor skew is OK, but no outliers. If outliers or significant skew, then same as above.
     * **n at least 40** - no need to display data. CLT will guarantee an approximately normal sampling distribution.

3. $\bar{x} \pm t^*\left(\frac{s}{\sqrt{n}}\right) = \bar{x} \pm m/e = (\_\_,\_\_)$

in context

4. I am ___% confident that the true mean ________ of __________ lies between _____ and _____.

Guide to Significance Test - unknown σ (1 sample t-test)

purpose parameter population

1. I want to test the claim that the true mean ___ of _____________ is (<,>,or≠) than the assumed truth. I suspect it is _____. (Creative license with this wording is OK as long as it includes Ho and Ha in words (including population and parameter) and then is followed with symbols.)

   Ho: μ = ___ Ha: μ (<,>,≠) ___

2. Since σ is unknown, I will perform a 1 sample t-test
   - **Condition 1**: need an unbiased sample
     * If SRS, then OK.
     * If random or not specified, then address the issue of potential bias in your results
   - **Condition 2**: need an approximately normal population
     * **n < 15** - display data, must be single peaked and approx. symmetrical. No skew or outliers. If there are, it must be mentioned as a major issue in generalizing your results to the population.
     * **n between 15 and 39** - display data, minor skew is OK, but no outliers. If outliers, then same as above.
     * **n at least 40** - no need to display data. CLT will guarantee an approximately normal sampling distribution.

3. $P(\bar{x}___) = P\left(t \begin{Bmatrix} > \\ < \end{Bmatrix} \frac{\bar{x}-\mu_o}{s/\sqrt{n}}\right) = P(t___) = $ p-value

   remember to double the probability for a 2 sided test.

4. With a p-value of p-value (connect p to α) I have strong enough evidence to reject Ho (p<α) **OR** I don't have strong enough evidence (p>α), so I fail to reject Ho and conclude in context.

Table entry for p and C is the point t^* with probability p lying above it and probability C lying between $-t^*$ and t^*.

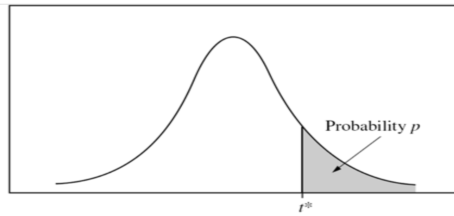


Table B *t* **distribution critical values**

df	Tail probability p											
	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	.703	.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	.690	.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	.689	.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	.688	.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	.688	.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	.687	.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	.686	.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	.686	.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	.685	.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	.685	.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	.684	.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	.684	.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	.684	.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	.683	.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	.683	.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	.683	.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	.681	.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	.679	.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	.679	.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	.678	.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	.677	.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	.675	.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
∞	.674	.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level C											