

9.3 Sample Means

The sampling distribution of a sample mean \bar{x} is for all possible srs of size = n are taken from a population with mean μ and standard deviation σ . Two key properties:

the mean of the sample mean \bar{x} is $\mu_{\bar{x}} = \mu$

the standard deviation of the sample mean \bar{x} is $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

Ex:

In a large HS of 2500 students, the mean number of cars owned by students' families is 2.35 with a standard deviation of 1.06. A SRS of 36 students is taken and the mean number of cars owned is calculated. What are the mean and standard deviation of the sample mean, \bar{x} ?

Population mean $\mu = 2.35$, $\sigma = 1.06$ cars, and $n = 36$.

the mean of the sample mean \bar{x} is $\mu_{\bar{x}} = \mu = 2.35$ and

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{1.06}{\sqrt{36}} \approx .177 \text{ cars}$$

Normal Distribution:

If the population distribution is normal, then so is the distribution of the sample mean.

$$N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$$

Use $z = \frac{X - \mu}{\sigma}$ to find the probability of actual values from the population

but

Use $z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}}$ to find the probability of a sample mean

Shape of Sampling Distributions:

When dealing with sampling distribution of sample proportions (\hat{p}), we were able to use a normal approximation for the distribution when np and nq are both at least 10, since it is essentially binomial in nature.

Central Limit Theorem

When dealing with sampling distribution of means (\bar{x}), the shape of the distribution will be the same as the population. If the population is normal, then we are all set. If the population is skewed, then the sampling distribution will be skewed unless the sample size is large enough. At a certain point, when you increase n , the sampling distribution will take on a more normal shape.



Attachments



Penny Applet



CLT applet