Refresh memory:

Binomial Distribution:

S success/failure

N fixed number "n" of observations

I each observation is independent

P each observation has same probability "p"

Binomial variable "X" counts number of successes (k) in the fixed number of trials

Examples:

How many free throws will Corinne make if she makes 75% of her throws and shoots 12 in a game?

How many children out of 5 in one family will have type "O" blood if there is a 25% chance of each child getting it?

8.2 Geometric Distributions

S success/failure

H how many observations until success?

I each observation is independent

P each observation has same probability "p"

Random variable "X" counts number of trials (n) needed to reach the first success.

Examples:

How many times will I need to flip a coin until I get a head?

How many times will I roll a die before getting a 2?

How many times will Corinne attempt a three point shot until she makes a basket?

*notice in each example, we are counting trials (n), not successes (k).

Example 8.15 (p.465)

An experiment has you rolling a single die. We want to roll a 2, which we will call a success. The random variable X = the number of trials until a 2 occurs.

Check for SHIP: (2 = success, all others = failure, how many rolls until a 2, die rolls are independent, each roll has a 1/6 probability)

X = 1: P(X=1) = P(success on 1st roll) = 1/6
X = 2: P(X=2) = P(success on 2nd roll)
= P(failure on 1st and success on 2nd)
= P(failure on 1st) x P(success on 2nd) *independent
= (5/6)(1/6)
X = 3: P(X=3) = P(F on 1st) x P(F on 2nd) x P(S on 3rd)
= (5/6)(5/6)(1/6)
$P(X=n) = (5/6)^{n-1}(1/6)$

General Rule for calculating Geometric Probabilities:

$$P(X=n) = (1-p)^{n-1}(p)$$

Creating a probability distribution table

use the dice experiment:

X	1	2	3	4
P(X)	1/6	(5/6)(1/6)	(5/6)(5/6)(1/6)	(5/6)(5/6)(5/6)(1/6)
P(X)	p	qp	q^2p	q^3p

with the calculator, just multiply each P(X) by P(failure)

The terms of P(X) are a geometric sequence, since each term is being multiplied by the same constant to get to the next term

You don't need to know this, but it's interesting...

Theoretically, to be a valid distribution, the probabilities have to add to 1.

$$S = p + (1-p)p + (1-p)^2p + (1-p)^3p + \ldots$$

let $1-p = q$

$$S = p + qp + q^2p + q^3p + \ldots$$

multiply both sides by q (because we can)

$$qS = qp + q(qp) + q(q^2p) + q(q^3p) + \ldots$$

$$qS = qp + q^2p + q^3p + q^4p + \ldots$$ (simplify)

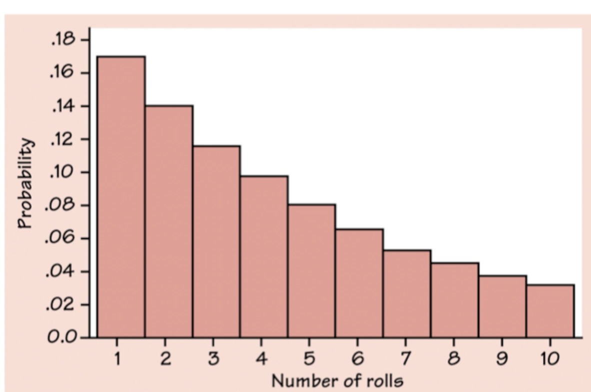
$$S = p + qp + q^2p + q^3p + \ldots$$ (remember what S= above)

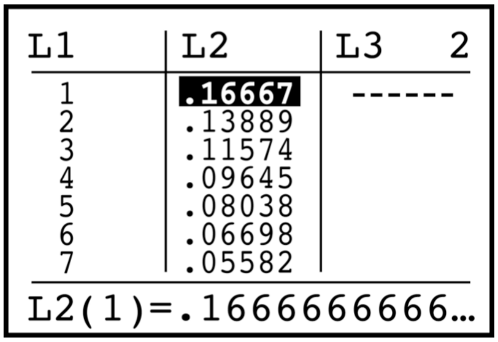
$S - qS = p$ (subtract the two equations)

$S(1-q) = p$ ... $S = p/(1-q)$ or $p/p = 1$ (work algebra magic)

Be able to graph a geometric distribution: (histogram)

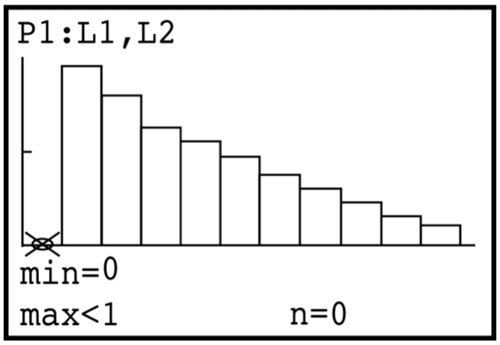
X	1	2	3	4
P(X)	1/6	(5/6)(1/6)	(5/6)(5/6)(1/6)	(5/6)(5/6)(5/6)(1/6)
or	.16667	.13889	.11574	.09645





Using the calculator:

Create L1 with values for X (use 1-10)
highlight L2 header and select:
2nd | VARS | geometpdf(1/6,L1)
and then hit enter.



Now generate a histogram with L1,L2

Calculating the mean (expected value)

X	1	2	3	4	...
P(X)	p	qp	q^2p	q^3p	...

$\mu_x = (1)(p) + (2)(qp) + (3)(q^2p) + (4)(q^3p) + \ldots$

$\mu_x = p + 2qp + 3q^2p + 4q^3p + \ldots$

$q\mu_x = qp + 2q^2p + 3q^3p + 4q^4p + \ldots$ multiply both sides by q:

$\mu_x - q\mu_x = p + qp + q^2p + q^3p + \ldots$ subtract the two equations

$\mu_x(1-q) = p + qp + q^2p + q^3p + \ldots$ sum of infinite series = 1

$\mu_x(p) = 1$

$\mu_x = 1/p$

$\mu_x = 1/p$ $\sigma_x^2 = q/p^2$ $\sigma_x = \sqrt{q}/p$

Example:

If the probability of success (rolling a 2) is 1/6, and we know the distribution is geometric (ex. 8.15), then the Expected value for number of rolls to get a 2 is:

$\mu_x = 1/p$
$= 1/(1/6) = 6$

You would expect to roll the dice 6 times to get a 2

If we want to know the probability that it takes more than 6 rolls to get a 2, then:

$P(X>6) = (1-p)^n$

$= (5/6)^6$

$\approx .335$

$P(X>n) = q^n$