2.1 Conditional Statements

If ________________, then ______________.
hypothesis (condition) conclusion

Example 1: Rearrange a sentence into if-then form

Flowers will not bloom if there is snow on the ground.

If there is snow on the ground, then flowers will not bloom.

Example 2: Put a sentence without an "if" into if-then form

All sharks have a boneless skeleton.

If an animal (or a fish) is a shark, then it has a boneless skeleton.

Conditional statements can be true or false.

A conditional statement is TRUE when the condition is met and the conclusion is true. It will be our job to prove these.

If two angles are right angles, then they are congruent.

A conditional statement is FALSE when at least one counterexample can show that when the condition is met, the conclusion is not true.

If a number is odd, then it is divisible by 3.

counterexample: 5

Original and Converse

Original: A conditional in if-then form.

If the measure of an angle is 30 degrees, then it is an acute angle.

Converse: You get by switching the hypothesis and conclusion.

If it is an acute angle, then the measure of an angle is 30 degrees.

but since that sentence doesn't flow well, we need to change it:

If an angle is acute, then its measure is 30 degrees.

Negation, Inverse and Contrapositive:

Negate: Change the statement so that it is the negative of what it was.

Original statement:	Negation:
$m\angle A = 30$	$m\angle A \neq 30$
$\angle$A is acute	$\angle$A is not acute

$\neq$ is not equal

The Inverse - when you negate **both** the hypothesis and conclusion of a conditional statement.

If the measure of an angle is **NOT** 30 degrees, then it is **NOT** an acute angle.

The Contrapositive - when you switch the negated hypothesis and conclusion of an inverse. (or when you negate a converse)

If it is **NOT** an acute angle, then the measure of an angle is **NOT** 30 degrees.

Important Postulates to know:

Postulate 5: Through any two points there exists exactly one line.

Postulate 6: A line contains at least two points.

Postulate 7: If two lines intersect, then their intersection is exactly one point.

Postulate 8: Through any three noncollinear points there exists exactly one plane.

Postulate 9: A plane contains at least three noncollinear points.

Postulate 10: If two points lie in a plane, then the line containing them lies in the plane.

Postulate 11: If two planes intersect, then their intersection is a line.

Bellwork:

Using the true conditional statement, "Constrictors (such as pythons) are not poisonous," write each of the following.

1. Rewrite the statement in if-then form.

   If a snake is a constrictor, then it is not poisonous.

2. Underline and identify the hypothesis and conclusion in #1.
3. Write the converse of the if-then statement.

   If a snake is not poisonous, then it is a constrictor.

4. Write the inverse of the if-then statement.

   If a snake is not a constrictor, then it is poisonous

5. Write the contrapositive of the if-then statement.

   If a snake is poisonous, then it is not a constrictor.

2.2 Definitions and Biconditional Statements

Goals: Learn about definitions and biconditional statements.

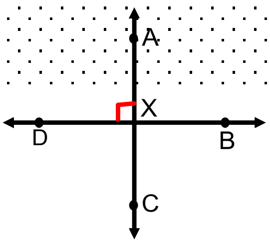
Definitions use words we know to describe new things. All definitions can be read forwards and backwards.

ex 1: definition of perpendicular (⊥): ⊥ is perpendicular to.

If two lines are perpendicular, then they intersect to form a right angle.

If two lines intersect to form a right angle, then they are perpendicular.

ex 2: use definitions to answer questions



a. Are points D, X and B collinear? Yes

b. Is $\overleftrightarrow{AC}$ ⊥ to $\overleftrightarrow{DB}$? Y

c. is ∠AXB adjacent to ∠CXD? No

"only if":

Another way to write a conditional statement:

ex 3:
If he committed a crime, then he was at the scene of the crime.

He committed a crime only if he was at the scene of the crime.
concl.

TRICKY - because the conclusion comes after the "if"

biconditional:

Combine conditional and converse in one sentence using "if and only if".
It can be true or false.

ex 4:
He committed a crime if and only if he was at the scene of the crime.
hyp./concl.

TRUE or FALSE?

All definitions can be written as a true biconditional statement.

Two lines are perpendicular if and only if they intersect to form a right angle.

ex: If $x^2 = 4$, then x = 2 or -2. (T)

Write the converse. T or F?

If x = 2 or -2, then $x^2 = 4$. (T)

If T, write the biconditional.

$x^2 = 4$ if and only if x = 2 or -2.

* write the biconditional in the same order as the original.

Bellwork:

Rewrite the biconditional statement as a conditional statement and its converse.

1. We will go to the beach if and only if it is sunny.

conv: If it is sunny, then we will go to the beach.

cond: If we go to the beach, then it is sunny.

Give a counterexample that demonstrates that the statement is false.

2. If a polygon has four equal sides, then it is a square.

a rhombus

3. If a vehicle has wheels, then it is a car.

bus, scooter, airplane,

Determine whether the statement can be combined with its converse to form a true biconditional.

4. If $2x > 8$, then $x = 5$.

can't be done.

2.3 **Deductive Reasoning**

Goals today: to learn symbolic notation for conditional statements AND to learn the two laws of logic.

If it is raining, then it is cloudy.

p (it is raining) q (it is cloudy)

p = hypothesis

q = conclusion

$\longrightarrow$ = then

~ = not

conditional (original) $p \longrightarrow q$ if it is raining then it is cloudy.	**converse** (switched) $q \longrightarrow p$ if it is cloudy, then it is raining.
inverse (negated) $\sim p \longrightarrow \sim q$ if it is not raining then it is not cloudy.	**contrapositive** (switched & negated) $\sim q \longrightarrow \sim p$ if it is not cloudy, then it is not raining.
biconditional (forwards and backwards) conditional + converse (if and only if) $p \longleftrightarrow q$ it is raining if and only if it is cloudy. $\longleftrightarrow$	

*The same colors always have the same truth values.

Example:

Let p be "today is Monday" and q be "there is school."

~p : today is NOT Monday. ~q : there is NO school

a. Write $p \rightarrow q$. If today is Monday, then there is school.

b. Write the converse of $p \rightarrow q$. $q \rightarrow p$ If there is school then today is Monday.

c. Write the contrapositive of $p \rightarrow q$. $\sim q \rightarrow \sim p$
If there is no school, then today is not Monday

d. Write the inverse of $p \rightarrow q$. $\sim p \rightarrow \sim q$
If today is not Monday, then there is no school

Two types of Reasoning

Inductive Reasoning vs. Deductive Reasoning

Inductive reasoning:	Deductive reasoning:
Patterns	Facts
Specific Examples	Rules
Prediction	Definitions
Conjecture	Logical Argument

Example: (Inductive or Deductive?)

inductive

A. Josh knows that Dell computers cost less than Mac computers. All other brands that Josh knows of cost less than Dell. Josh reasons that Mac costs more than all other brands.

B. Josh knows that Dell computers cost less than Mac computers. He also knows that Mac computers cost less than Micron. Josh reasons that Dell costs less than Micron.

deductive

Two Laws of Logic (Law of Detachment and Law of Syllogism):

When you have **one** conditional statement ($p \longrightarrow q$) and it is **true**...

you can "detach" the p and if the condition is met, you can draw the conclusion. It's called the **Law of Detachment**.

Example 1:

If x is a number divisible by 5, then x must end in a 0 or a 5.
x = 25

25 ends in a 5.

Example 2:

If today is Labor day, then there is no school.
Today is Labor day. there is no school

It only works when "p" is the part that is detached!

can't detach the q

When you have **two** conditional statements ($p \longrightarrow q$ and $q \longrightarrow r$) and they are **both true**...

as long as the conclusion of the first statement is the same as the hypothesis for the second statement you can leap from p to r ($p \longrightarrow r$) and it is true. It's called the **Law of Syllogism**. (with logic)

Example:

If you attend Olympia High School, then you live in Orlando.
p q

If you live in Orlando, then you live in Florida.
q r

so you can conclude:

If you attend Olympia High School, then you live in Florida.
p r

Example:

Heather is going to the mall. Can you conclude that she will buy a pair of shoes?

If Heather shops for shoes, she will buy a pair. If Heather goes to the mall she will shop for shoes.

switch the two sentences and yes, she will buy shoes!

2.4 Reasoning With Properties from Algebra

Equations: Whatever you do to one side - you have to do to the other

(+ prop =) **Addition** (− prop =) **Subtraction**

(× prop =) **Multiplication** **Division** (÷ prop =) of Properties of Equality

Other properties from Algebra:

Reflexive: looks the same on both sides of the equation
a = a AB = AB m∠A = m∠A (reflection)

Symmetric: Can flip the equation over (symmetry)
If a = b, then b = a

Transitive: If one thing equals another and that equals a third, then the first thing equals the third thing (transit (to move))
If a = b and b = c, then a = c

Substitution: If two things are equal, then you can replace one for the other

Distributive: a(b + c) = ab + ac

simplify: combining like terms

We need to be able to solve algebra equations and justify each step along the way:

Ex 1:

Statements (Reasons)

3x + 12 = 8x - 18 (Given)

−3x −3x

12 = 5x − 18 (− prop =)

+18 +18

36 = 5x (+ prop =)

÷5 ÷5

6 = x (÷ prop =)

x = 6 (symmetric)

Ex 2:

Statements (Reasons)

55z - 3(9z +12) = -64 (Given)

55z - 27z - 36 = -64 (distributive)

28z - 36 = -64 (Simplify)

+36 +36

28z = -28 (+ prop =)

28 28

z = -1 (÷ prop =)

Examples:

Addition property of equality: If $AB = 5$, then $10 + AB =$ ___?___ 15.

Multiplication property of equality: If $m\angle C = 30°$, then ___?___ 5 $(m\angle C) = 15°$.

Reflexive property of equality: $AF =$ ___ AF.

Symmetric property of equality: If $m\angle DCF = m\angle MJC$, then ∠MJC = ∠DCF

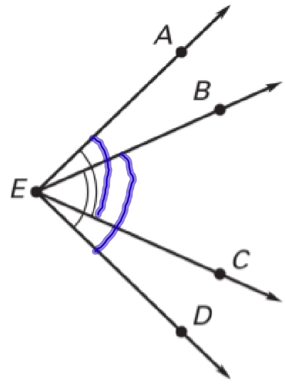
Transitive property of equality: If $YZ = DB$ and ___ DB $= JK$, then ___ YZ = JK

Substitution property of equality: If $MN = 3$, then 5(MN) 3 = ___ 15

m∠C = 30

Now with Geometry

Ex:



$m\angle AEB + m\angle BEC = m\angle CED + m\angle BEC$ (Given)

$- \; m\angle BEC = m\angle BEC$ (reflexive)

$m\angle AEB = m\angle CED$ (− prop =)

Ex:

$AB = BC$ (Given)

$AC = AB + BC$ (Segment + post)

$AC = AB + AB$ (substitution prop)

$AC = 2(AB)$ (distributive prop)

Bellwork:

Use the property to complete each statement.

1. Multiplication property of equality: If $x = y$, then $ax =$ ay.

2. Symmetric property of equality: If $FG = HJ$, then HJ = FG.

3. Transitive property of equality: If $BC = KL$, and $KL = YZ$, then BC = YZ

Solve the equation and state a reason for each step.

4. $3m - 2 = m + 8$ (Given)

$-m \qquad -m$

$2m - 2 = 8$ (− prop =)

$+2 \quad +2$

$2m = 10$ (+ prop =)

$\frac{2m}{2} = \frac{10}{2}$

$m = 5$ (÷ prop =)

2.5 Proving Statements about Segments

Goal: Learn how to write a proof

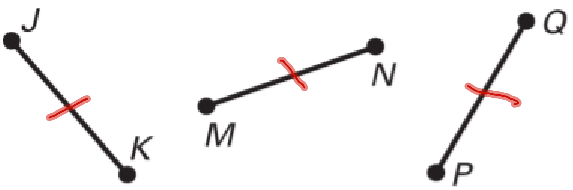
theorem: a true statement that can be proven using deductive reasoning

THEOREM 2.1 PROPERTIES OF SEGMENT CONGRUENCE	
Reflexive	For any segment AB, $\overline{AB} \cong \overline{AB}$.
Symmetric	If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$
Transitive	If $\overline{AB} \cong \overline{CD}$, and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$

writing proofs:

- definitions (definition of congruent, perpendicular)
- postulates (angle addition, segment addition)
- theorems

Ex 1: 2 column proof

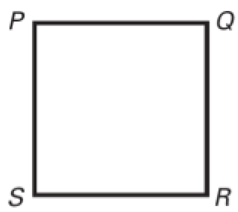


Given: $\overline{JK} \cong \overline{MN}$, $\overline{MN} \cong \overline{PQ}$
Prove: $\overline{JK} \cong \overline{PQ}$

Statements	Reasons
1. $\overline{JK} \cong \overline{MN}$, $\overline{MN} \cong \overline{PQ}$	1. Given
2. $JK = MN$, $MN = PQ$	2. def ≅
3. $JK = PQ$	3. Transitive property of equality
4. $\overline{JK} \cong \overline{PQ}$	4. Definition of congruent segments

Ex 2:

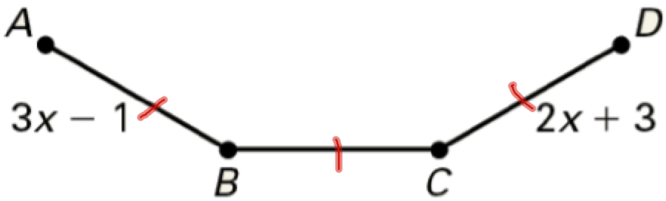
Given: $\overline{PQ} \cong \overline{RS}$, $\overline{PQ} \cong \overline{QR}$, $\overline{PS} \cong \overline{RS}$
Prove: $\overline{PS} \cong \overline{QR}$



Statements	Reasons
1. $\overline{PQ} \cong \overline{RS}$	1. Given
2. $\overline{PQ} \cong \overline{QR}$	2. Given
3. $\overline{RS} \cong \overline{QR}$	3. Transitive Property of Congruence
4. $\overline{PS} \cong \overline{RS}$	4. Given
5. $\overline{PS} \cong \overline{QR}$	5. Transitive Property of Congruence

Ex 3:

In the diagram, if $\overline{AB} \cong \overline{BC}$ and $\overline{BC} \cong \overline{CD}$, find BC.



$\overline{AB} \cong \overline{BC}$, $\overline{BC} \cong \overline{CD}$ (Given)

$\overline{AB} \cong \overline{CD}$ (Transitive prop ≅)

$AB = CD$ (def ≅)

$3x - 1 = 2x + 3$ (substitution)

$x - 1 = 3$ (− prop =)

$x = 4$ (+ prop =)

$BC = 11$ (substitution)

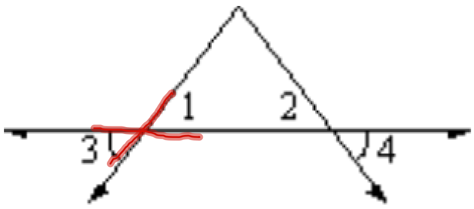
$3x - 1$
$3(4) - 1$
$12 - 1$
11

$2(4) + 3$
$8 + 3$
11

Bellwork:

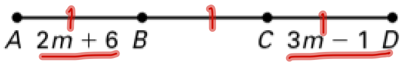
Given : ∠3 ≅ ∠4

Prove : ∠1 ≅ ∠2



Statement	Reason
1. ~~∠3 ≅ ∠4~~	a. Given
2. ~~∠1 ≅ ∠3;~~ ∠4 ≅ ∠2	b. Vertical Angles
3. ∠1 ≅ ∠4	c. Transitive Property of Congruence
4. ∠1 ≅ ∠2	d. Transitive

Solve for the variable using the given information. Explain your steps.



Given $\overline{AB} \cong \overline{BC}$, $\overline{CD} \cong \overline{BC}$

→ $\overline{AB} \cong \overline{CD}$ (transitive)

→ $2m+6 = 3m-1$ (def ≅)

$-2m \quad -2m$

$6 = m-1$ (− prop)

$+1 \quad +1$

$7 = m$ (+ prop)

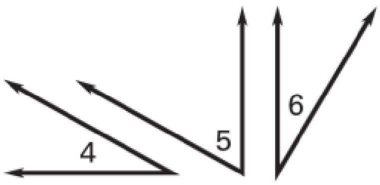
$m = 7$ (symmetric)

2.6 Proving Statements about Angles

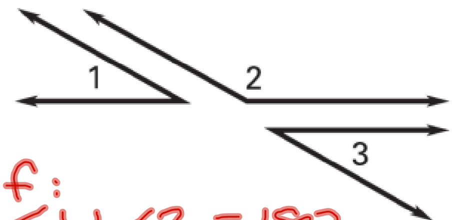
Properties of Angle Congruence

Reflexive	For any angle *A*, ∠A ≅ ∠A
Symmetric	If ∠*A* ≅ ∠*B*, then ∠B ≅ ∠A
Transitive	If ∠*A* ≅ ∠*B* and ∠*B* ≅ ∠*C*, then ∠A ≅ ∠C.

Congruent Complements and Supplements

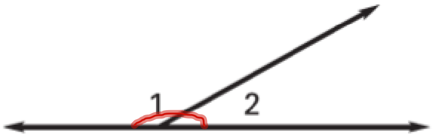


if:
∠4 + ∠5 = 90
∠6 + ∠5 = 90
then:
∠4 ≅ ∠6



if:
∠1 + ∠2 = 180
∠3 + ∠2 = 180
then:
∠1 ≅ ∠3

Linear Pair Postulate = Linear pair ∠s are Supplementary

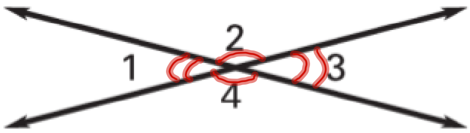


$\angle 1 + \angle 2 = 180$

add them up:

Vertical Angle Theorem

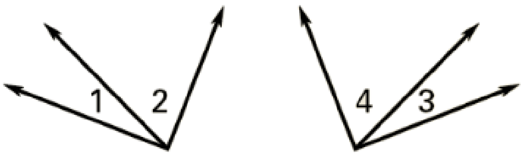
set them equal:



$\angle 1 = \angle 3$

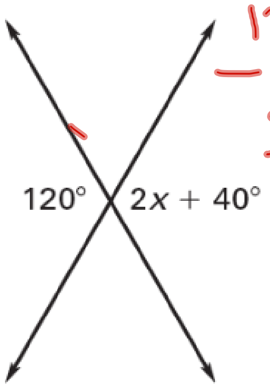
$\angle 2 = \angle 4$

Ex 1:



Given: $m\angle 1 = 24°$, $m\angle 3 = 24°$,
$\angle 1$ and $\angle 2$ are complementary,
$\angle 3$ and $\angle 4$ are complementary
Prove: $\angle 2 \cong \angle 4$

Ex 2:



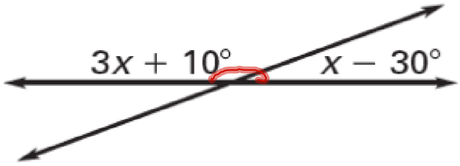
$120 = 2x + 40$ (vertical angles)
$-40 \quad\quad -40$

$\frac{80}{2} = \frac{2x}{2}$ (− prop =)

$40 = x$ (÷ prop =)

$x = 40$ (symmetric)

Ex 3:



$3x + 10 + x - 30 = 180$ (Linear pair)

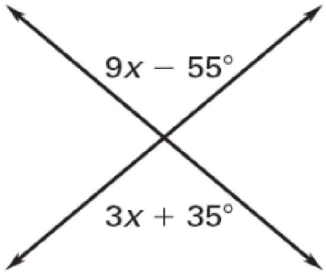
$4x - 20 = 180$ (simplify)

$+20 \quad +20$

$\frac{4x}{4} = \frac{200}{4}$ (+ prop)

$x = 50$ (÷ prop)

Ex 4:



$9x - 55 = 3x + 35$ (vert ∠)

$-3x \quad -3x$

$6x - 55 = 35$ (− prop)

$+55 \quad +55$

$\frac{6x}{6} = \frac{90}{6}$ (+ prop)

$x = 15$ (÷ prop)