

Chapter 2: The Normal Distributions

2.1 Density Curves and Normal Distributions

- Density Curves are how we display distribution for whole population (histogram, boxplot, stemplot, dotplot - all for samples)
- Area in a density curve represents proportion of population
- median divides density curve into equal areas
- mean divides density curve into equal "weights" (balancing point)
- 68-95-99.7 rule (empirical rule) for normal distributions

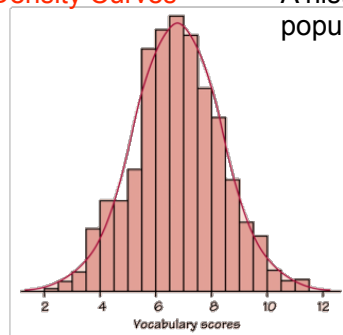
2.2 Standard Normal Calculations

- z-scores are a measure of how many standard deviations away from the mean a value in the data lies
- use z-score to find proportion and use proportion to find z-score
- difference between \bar{x} and μ as well as "s" and σ
- standard normal distribution: $N(0,1)$
- use table A and calculator to find proportions
- find points in data where all values fall above or below
- be able to assess whether a distribution is normal or not
- normal probability plots

2.1 Density Curves and the Normal Distribution

Density Curves

A histogram represents a sample of data from a population we are interested in



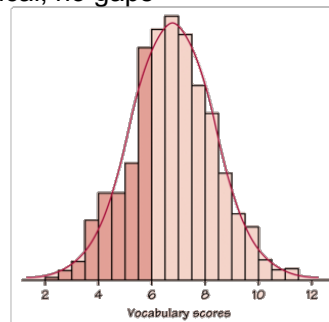
A density curve smooths out the bumps and allows us to use area to calculate proportions

- 947 Indiana 7th graders
- Iowa Test of Basic Skills
- symmetrical, no gaps

The darker bars represent 287 students from the data set.

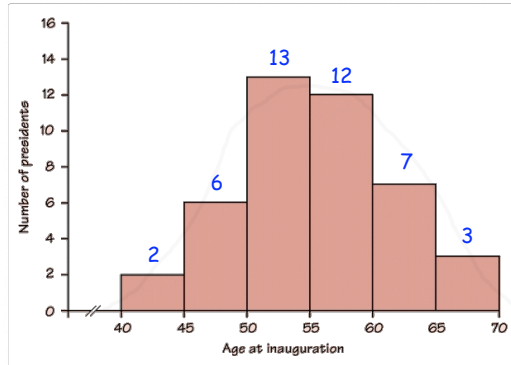
$287/947 = .303$
30.3% of the students scored less than or = 6

All the bars represent $947/947 = 1$
so 100% of the sample is represented by the bars



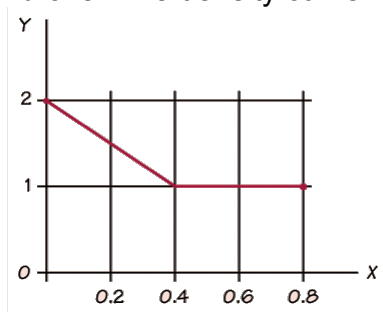
If we calculated the area under the curve (ignoring the bars) it would be .293 or 29.3% - VERY close to the actual percent.

Let's try using our Prez data



There are 43 observations in this data. What percent of the observations fall at or below age 50?

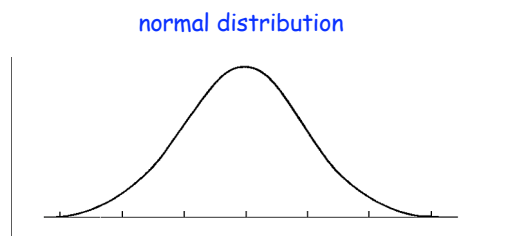
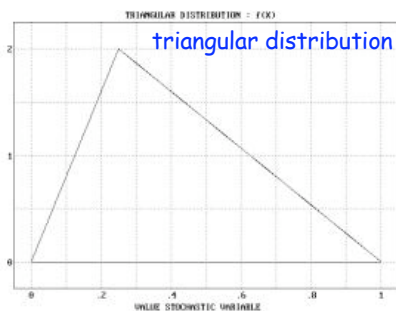
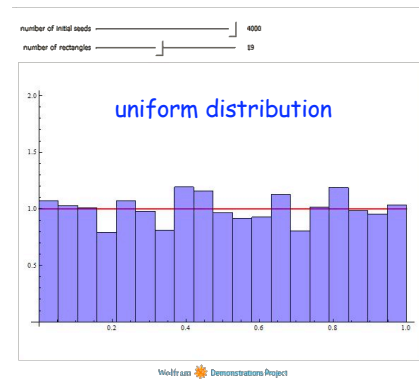
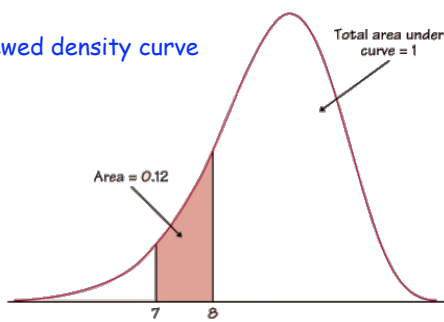
A broken line density curve



Things to know about all density curves:

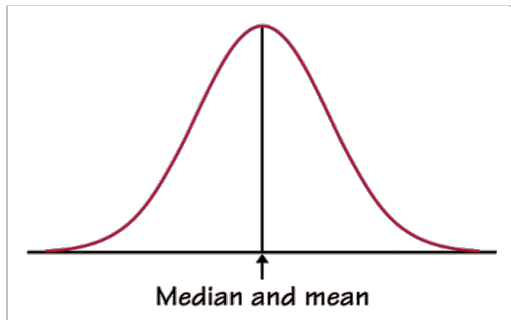
- The **total area** under the curve always = 1
- Always on or just above the horizontal axis

left skewed density curve

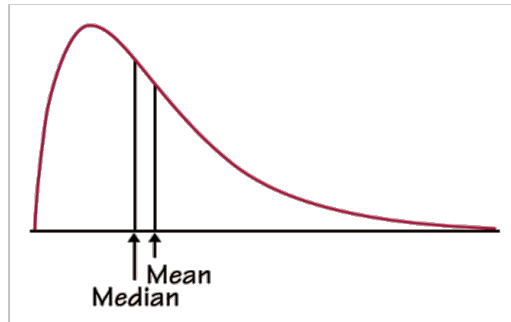


How mean and median behave in a density curve

When the data is normally distributed, the curve is symmetrical and the mean = median



When the distribution is skewed right, the mean > median *



* For a left skewed distribution mean < median

Important to note:

median - divides a density curve into two equal areas

mean - balancing point of the density curve if it were weighted according to the values in the distribution

What you need to know about the symbols:

For a set of data displayed graphically:

histogram	stemplot
boxplot	dotplot

we use the actual observations to calculate:

\bar{x} for mean

S for standard deviation

for a density curve which is an idealized mathematical model we use:

μ for mean

σ for standard deviation

The Normal Curve

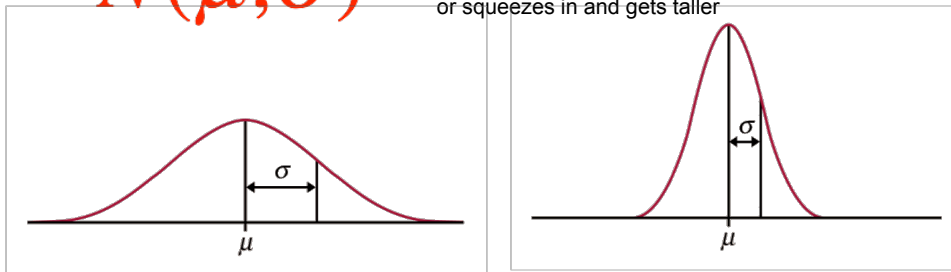
Density curves that are
 symmetric
 single peaked (unimodal)
 bell shaped

They describe normal distributions, all of which have the same overall shape

Normal distributions, since they are symmetric are described using mean and standard deviation. When we use the Normal curve we use the following notation:

$N(\mu, \sigma)$

If mean changes, graph slides left or right
 If std. dev. changes, graph spreads out and gets shorter or squeezes in and gets taller



use points of inflection to estimate standard deviation

The Empirical Rule (68/95/99.7)

Empirical Rule applet

All normal density curves satisfy the following property (the Empirical Rule).

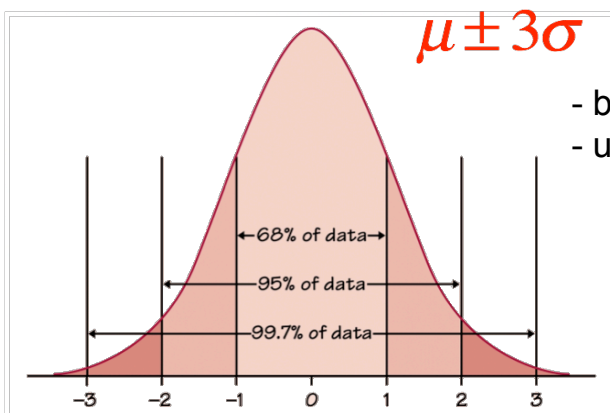
68% of the observations fall within 1 standard deviation (sd) of the mean*

$$\mu \pm \sigma$$

95% of the observations fall within 2 standard deviations of the mean

$$\mu \pm 2\sigma$$

99.7% of the observations fall within 3 standard deviations of the mean



- break down the tail %'s:
- use percentiles with a normal curve

In real life data a distribution is rarely "exactly" normal, but many are approximately normal and satisfy the Empirical rule closely enough for regular use.

Practice:

given a distribution that is $N(64.5, 2.5)$ of young women's heights:

- find the spread of heights for the middle 95% of young women
- what percent of young women have a height above 69.5?

