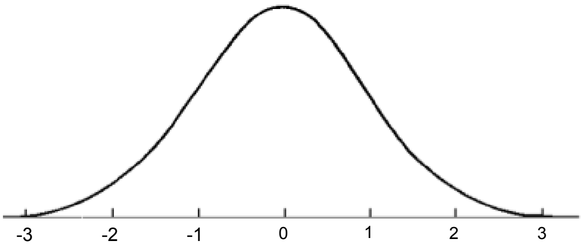
2.2 Standard Normal Calculations

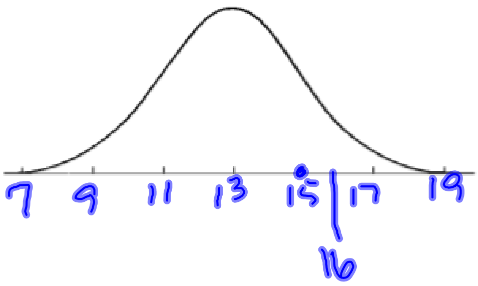
standardized score is called a z-score

- converts any observation into how many standard deviations (σ) an observation is away from its mean (μ)
- the z-score formula: $$z = \frac{x-\mu}{\sigma}$$
- when you standardize, the mean ($\mu$is always "0" σway from the mean
- the standard normal distribution is N(0,1)
- we standardize in order to compare distributions that are not alike
- a standardized normal curve will always look like this:

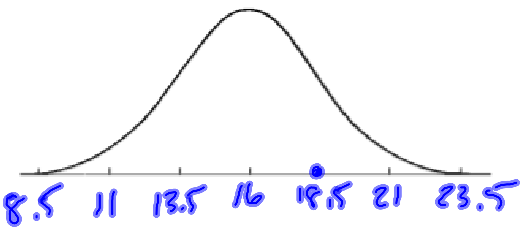


Example: we want to compare students from one school to students from a school in another county. Both took a 10th grade math benchmark test (one was 20 points and one was 25 points) and each group was normally distributed as so:

School A: N(13,2)



School B: N(16,2.5)



If a student from school A scored 15 and a student from school B scored 18.5, how do they compare?

$$z = \frac{x-\mu}{\sigma} = \frac{15-13}{2} = 1$$

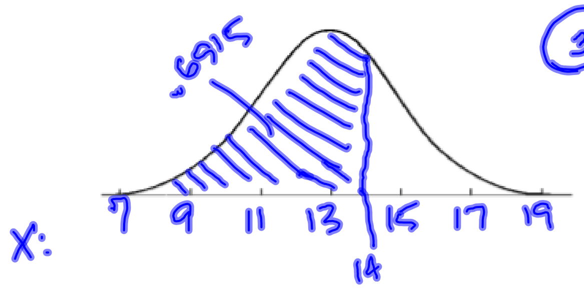
$$z = \frac{18.5-16}{2.5} = 1$$

How do you find the percent of students from school A who scored less than 14?

Do 4 things:
1. rewrite in terms of x
2. calculate z-score, draw & shade
3. use the table
4. write your conclusion in context

① $x < 14$

② $z = \frac{x-\mu}{\sigma} = \frac{14-13}{2} = .5$

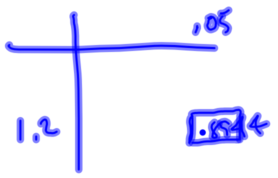


③ area $z < .5 = .6915$

④ The percent of students from School A who scored less than 14 is 69.15%.

always check to make sure your answer makes sense with the empirical rule!

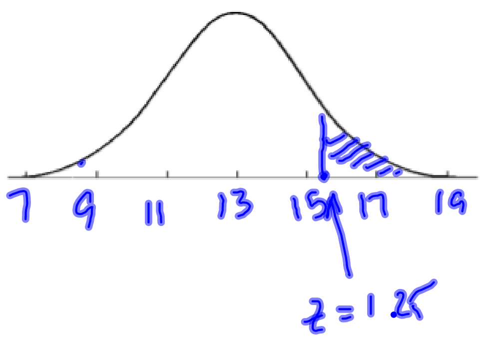
Repeat, but calculate the percent of students who scored above 15.5



① $X > 15.5$

② $z = \frac{x-\mu}{\sigma} = \frac{15.5-13}{2}$

$= \frac{2.5}{2} = 1.25$



③ area $z < 1.25 = .8944$

area $z > 1.25 = 1 - .8944$

$= .1056$

④ 10.56% of students @ school A scored above 15.5.

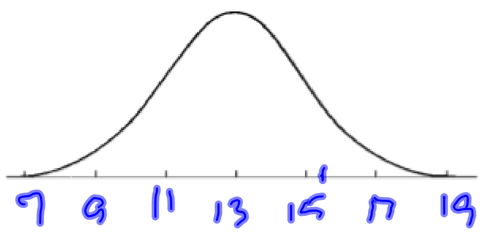
watch out for inequalities and shading of your curve!

Find a value in the data, given a proportion (use the table backwards)

using the same example, if a student scores at the 92nd percentile (%-ile), what score have they made?

$\text{area} = .92$

$.9207 \Rightarrow z = 1.41$



Do 3 things:
1. State the problem and sketch
2. Use the table backward
3. Unstandardize
   (use z formula to solve for x)
4. Write conclusion in context

$$z = \frac{x-\mu}{\sigma}$$

$$1.41 = \frac{x-13}{2}$$

$$2(1.41)+13 = x$$

$$15.82 = x$$

Assessing Normality (How to tell if a sample is normally distributed)

Two main ways to check for this:

1. Plot your data and check for symmetry and bell shape
   (it helps to calculate s and scale the axis with mean and s)

2. Construct a Normal Probability Plot (NPP)
   - enter data into a list on your TI-84
   - 2nd | y= (stat plot)
   - make sure plot 1 is on and the others are off
   - choose the bottom right type (directly under the histogram)
   - choose Data List: L1 if that is where you entered your data
   - choose Data Axis: you can use X or Y here
   - ZOOM 9
   - the straighter the line, the more normal the data
   - a NPP does NOT show a graphical display of the data...it is only a tool for assessing normality

Now do all three types of problem using the calculator instead of the table:

have z-score, want proportion? normalcdf(LB,UB)

the calculator assumes the standard normal curve N(0,1) and returns a standardized value (a z-score)

LB = Lower Boundary
UB = Upper Boundary
these mark the interval for which you want the area

have x value, want proportion? normalcdf(LB,UB, μ,σ)

skips the step of finding a z-score if you enter the mean and std. dev. of the population

have proportion (or %-ile), want z-score? invNorm(area)

working the table backwards, you enter the proportion/area and the calculator returns the z-score

have proportion (or %-ile), want x value? invNorm(area, μ,σ)

like working the table backwards, but if you specify the mu and sigma it will return the value from the data

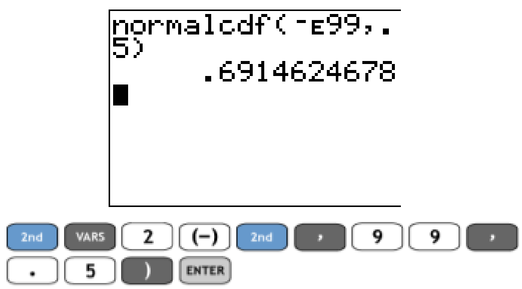
Practice with our example: N(13,2)

Proportion below 14:

x<14

calculate z-score:
z = (14-13)/2 = .5

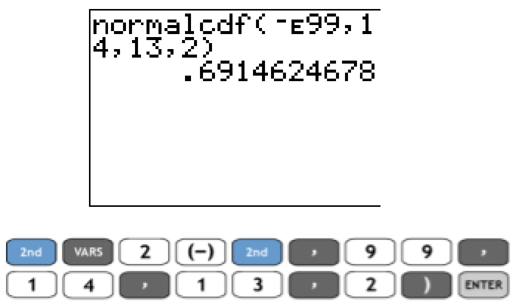
use (-∞,.5) as boundaries



Proportion below 14:

x<14

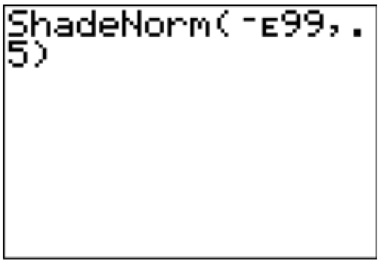
skip the z-score and use values and parameters from the data

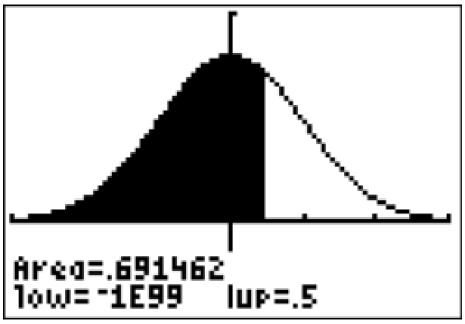


Shadenorm:

Shadenorm does everything that normalcdf does, but with a shaded graph to go along with it!

Before you do this, change your window settings to: X[-3,3]$_1$ and Y[-0.2,0.5]$_{0.1}$





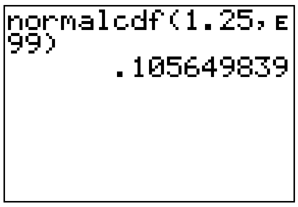
You can also use all 4 parameters: ShadeNorm(-E99,14,13,2) for the same result!

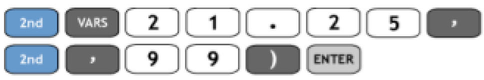
Capturing the area above a value:

Same example: N(13,2)

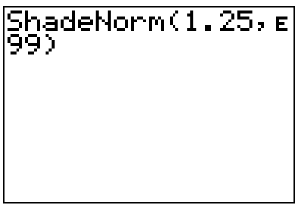
x > 15.5

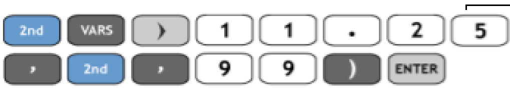
z = (15.5 - 13)/2 = 1.25

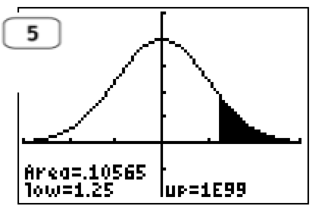




Notice, since our interval goes FROM 1.25 to +∞, we do NOT have to subtract from 1. You are telling the calculator what area you want and it returns that exact area.





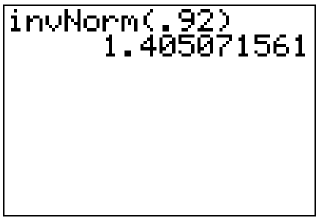


try again, without converting to a z-score: normalcdf (15.5,E99,13,2) and/or shadenorm (15.5,E99,13,2)

Working backwards:

From the same example N(13,2), what score would be in the 92nd percentile?

invNorm (.92) returns the z-score of about 1.4



1.4 = (x - 13)/2
2(1.4) + 13 = x
15.8 = x

A score of 15.8 would be in the 92nd percentile

If you enter the area/percentile along with the mu/sigma of the distribution, you skip the "unstandardizing" part of the problem.